Dynamic algebra as a constructive semantics for classical existential proofs

Mathematical Logic and Constructivity:
The Scope and Limits of Neutral Constructivism
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My favorite quote

Henri Poincaré, in *La logique de l'infini* (Revue de Métaphysique et de Morale, 1909).
As for me, I would propose that we be guided by the following rules:

1. Never consider any objects but those capable of being defined in a finite number of words;
2. Never lose sight of the fact that every proposition concerning infinity must be the translation, the precise statement of propositions concerning the finite;
3. Avoid nonpredicative classifications and definitions.

Books in constructive algebra after Bishop

Ray Mines, Fred Richman et Wim Ruitenburg. *A course in constructive algebra.* Universitext. Springer-Verlag, New York, (1988)
Harold M. Edwards. *Divisor theory.* Birkhäuser Boston, Inc., Boston, MA, (1990)
Harold M. Edwards. *Essays in constructive mathematics*, Springer, (2005)
Henri Lombardi et Claude Quitté. *Commutative algebra : constructive methods. Finite projective modules.* English translation, with additions and corrections, of the french book (Calvage & Mounet, Paris, 2011) Springer, (2015). https://arxiv.org/abs/1605.04832
Ihsen Yengui. *Constructive commutative algebra : projective modules over polynomial rings and dynamical Gröbner bases.* Springer Lecture Notes in Mathematics 2138. (2015)

Works directly related to this talk

About a new method for computing in algebraic number fields. Jean Della Dora, Claire Dicrescenzo and Dominique Duval. LNCS 204, Springer, (1985)
Dynamical method in algebra: Effective Nullstellensätze. Coste, L-, Roy. Annals of Pure and Applied Logic 111, (2001)
https://arxiv.org/abs/1701.05794,
A logical approach to abstract algebra. Coquand, L-. Math. Struct. in Comput. Science 16, (2006)
http://hlombardi.free.fr/publis/AlgebraLogicCoqLom.pdf
Structures algébriques dynamiques, espaces topologiques sans points et programme de Hilbert. L-. Ann. Pure Appl. Logic 137 (2006).
http://hlombardi.free.fr/publis/W2FTOP.pdf
a draft: *Théories Géométriques pour l'algèbre constructive.*
http://hlombardi.free.fr/Theories-geometriques.pdf

First order dynamical theory

A purely computational, logic free, version of a coherent theory

Dynamical algebraic structures

page 7

Collapsus versus classical existence theorem

page 8

Extensions that prove the same facts

Extensions that prove the same Horn rules

page 9

Two fundamental conservativity results

page 10

Essentially equivalent extensions

page 11

Spectra in Algebra

page 12

Infinitary geometric theories