

LOGIKSEMINARIET STOCKHOLM–UPPSALA

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I. Small hom-sets in the category of formal spaces and the Stone–Čech compactification

The category of formal spaces/locales, as considered in a predicative setting, is not locally small: there are classes of continuous functions between two given objects (hom-sets) that do not form a set. I will first recall results, due to Peter Aczel, Erik Palmgren, and myself, that identify some typologies of small hom-sets. The relation of these results with the existence of Stone–Čech compactification of a formal space S (the compact completely regular reflection of S) is then discussed: a family of compactifications is presented that contains in particular the Stone–Čech compactification of those spaces S for which the class of $[0, 1]$ -valued continuous functions on S is a set. It is also shown that the existence of Stone–Čech compactification of a formal space S is actually equivalent to this (set-theoretic) assumption. Stone–Čech compactification itself can be used to set-index particular kinds of hom-sets.

II. Metric and uniform formal spaces

The way-below, well and really inside relations, used in the point-free formulation of local compactness, regularity and complete regularity, respectively, may be regarded as ways of expressing formally the idea that a given (basic) neighbourhood is ‘finer than’, or is a ‘better approximation than’, another (basic) neighbourhood. By basing on the notion of elementary diameter, one may formulate quantitatively this concept in the form of the measurably and uniformly inside relations; these relations allow us to express metrisability and uniformizability for formal spaces. Elementary diameters can also be used to define natural categories of metric and uniform formal spaces. In this point-free setting, a fully general uniform continuity theorem can be proved constructively, using the point-free equivalent of the standard technique. These notions also lead to more informative representations of classes of points as sets: first, they allow us to regard points as ‘shrinking’ sequences (or nets), providing arbitrary precise approximations to the given point; secondly, these lead to prove that, if a locally compact or complete metric formal space is proper, then not only its points form a set, but also this set is inhabited (the theory describing the space has an effective model).

I anslutning till seminariet, ca kl. 11.30, ger
Erik Palmgren några kommentarer om åtskillnadsrum
(eng. *apartness spaces*) och formella topologier.

Onsdag 11 februari kl. 9.30–12.00,
sal 6002 (hus 6, samma ingång som restaurang Rullan),
MIC, Polacksbacken, Uppsala.