Logikseminariet Stockholm–Uppsala

Steve Vickers
(Birmingham)

Classifying categories

The notion of classifying category (or syntactic category, or theory category) has long been a central part of categorical logic. Suppose L is a logic – by which I mean, approximately, a selection of connectives and deduction rules. We assume a corresponding class CL of categories is understood, with the structure needed to intepret the connectives of L. Thus if T is any theory in L, and C is a CL-category, we can say what are the models of T in C.

A classifying category for T (with respect to L) is formed by freely building a CL-category on a formal "generic" model of T. Then T models in C can be extended, uniquely up to isomorphism, to functors from CL to C that preserve the CL-structure. A well known example is the classifying topos, where L is geometric logic.

A common construction (as "syntactic category") starts with a category whose objects and morphisms are equivalence classes of formulae. Structural inductions on formulae are then needed to prove the universal property of the classifying category.

My work with Erik Palmgren on Cartesian theories focuses attention on the use of the initial algebra theorem to embody structural inductions. We have applied this to the construction of classifying categories for the case where L is Cartesian logic (or more precisely for us quasi-equational logic) and CL is the class of Cartesian categories, with canonical terminal object and pullbacks. Corresponding syntactic categories are described in Johnstone's Elephant D, and analogous sketch-based constructions ("left-exact theories") are described in Barr and Wells' "Toposes, Triples and Theories". We show that for each quasi-equational theory T, there is another one, which we call Cart-with-T, whose models are cartesian categories equipped with models of T. Then the classifying category for T with respect to cartesian logic is constructed as an initial algebra for Cart-with-T. We prove its universal property in a strong form: the category of models of T in C is isomorphic (not just equivalent) to the category of strict cartesian functors from the classifying category to C. We can then relax this to an equivalence to cover the case where C does not have canonical finite limits.

It seems clear that the methods will also apply to other logics (though not to geometric logic because of the inifinitary disjunctions).

Onsdag 21 september kl. 10.30–12.15,
sal 3513 (hus 3), MIC,
Polacksbacken, Uppsala.

http://www.math.su.se/~jesper/seminarier/